STAISTICS – CHAPTER 5 NOTES

Binomial experiment (Bernoulli) – a class of problems that are characterized by the feature that are exactly two possible outcomes

1. There are a fixed number of trials (n)
2. The trials are independent and repeated under identical conditions.
3. Each trial has only two outcomes: success (S) and failure (F).
4. For each individual trial, the probability of failure (q). Note that probability of success (p) and the probability of failure (q). Note that $p+q=1$ and $1-p=q$.
5. The central problem of a binomial experiment is to find the probability of r success out of n trials.

Independent variables – the outcome of one trial cannot affect the outcome of any other trial. Any time a selection is made from a population without replacement, the trials are **not** independent.

General Formula for the binomial Probability Distribution- Table 4

$$P(r) = C_{n,r} p^r q^{n-r}$$

Optional (table 3)

The formula for the computation of the binomial coefficient $C_{n,r}$ is

$$C_{n,r} = \frac{n!}{r!(n-r)!}$$

where n! (read, *n factorial*) is the product of n with all the counting numbers less than n. 0! Is defined to be 1.

Mutually exclusive events – when you want to compute the probability that “at least” a certain number of events have occurred, rather than an exact amount, use the addition rule to find P(r).

Example: To find the probability of at least 5 successes out of 12 trials.

P(at least 5 successes)= $P(r \geq 5)$

$=P(r=5 \text{ or } 6 \text{ or } 7 \text{ or } 8 \text{ or } 9 \text{ or } 10 \text{ or } 11 \text{ or } 12)$

$= P(5) + P(6) + P(7) + P(8) + P(9) + P(10) + P(11) + P(12)$

Additional Properties -

Whenever p equals 0.5, the graph of the binomial distribution will be symmetrical no matter how many trials here are.

The **mean** μ is also the **expected value**.

For the binomial Distribution:

Mean $\mu = np$

Standard deviation $\sigma = \sqrt{npq}$

Where n is the number of trials, p *is the probability of success,* q is the probability of failure ($q = 1 - p$).

Geometric Probability Distribution

$$p(n) = p(1-p)^{n-1}$$

Where *n* is the number of the trial on which the first success occurs (n = 1, 2, 3, …) and *p* is the probability of success on each trial. *Note: p* must be the same for each trial.

Poisson Distribution

Let λ (Greek letter lambda) be the mean number of successes over time, volume, area, and so forth. Let *r* be the number of successes (r = 0, 1, 2, 3,….) in corresponding interval of time, volume, area, and so forth. Then the probability of *r* success in the interval is

$$P(r) = \frac{e^{-\lambda}\lambda^{r}}{r!}$$

Where *e* is called *Euler's constant* and is approximately equal to 2.7183

Poisson Approximation – this is for use as a distribution for "rare" events.

Consider the binomial distribution with

n = number of trials
r = number of successes
p = probability of success on each trial

If *n* ≥ 100 and *np* < 10, then *r* has a binomial distribution that is approximated by a Poisson distribution with $\lambda = np$.